Topological aspects of periodic and aperiodic photonic crystals

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ABSTRACT

Topological properties of Bloch modes have been demonstrated in purely dielectric photonic crystals at high symmetry points where a hidden time reversal-like symmetry can be exhibited. So far, the topological properties that have been shown are essentially due to these symmetries. For one dimensional structure, we define a topological invariant that can be extended to lossy or aperiodic structures. .

1. INTRODUCTION

This study takes place within the field of topological metamaterials.[1,2] The existence of edge states in one dimensional structures, e.g. stratified media, can be characterized by a topological invariant, when the structures have inversion symmetry. This is linked to the Zak phase.[3,4] The topological invariant can be expressed in terms of the poles and zeros of a meromorphic function. This allows to extend the notion to media with losses and also aperiodic media. The extension to higher dimensional structures is sketched. Numerical examples for one dimensional structures are given.

2. TOPOLOGICAL INVARIANTS OF PERIODIC STRUCTURE IN ONE DIMENSION

A non-trivial Berry phase can be associated to, say, a stratified medium, provided that the permittivity function presents an inversion symmetry.[3,4] Let us denote the permittivity by $\varepsilon(x)$. The medium under study is periodic with period d: $\varepsilon(x) = \varepsilon(x + d)$. We assume that there is an origin such that $\varepsilon(x) = \varepsilon(-x)$.

The determination of the field inside the medium is made by specifying the value of the field and its derivative at any given point x_0:

$$\mathcal{U}(x_0) = (u(x_0; k), u'(x_0; k)). \tag{1}$$

From this, the value at any other point is obtained by means of the resolvent $\mathcal{R}(x, x_0)$:

$$\mathcal{U}(x) = \mathcal{R}(x, x_0)\mathcal{U}(x_0). \tag{2}$$

The Bloch waves are given by[5,6,8,9]

$$u(x; k)e^{ikx},\ k \in [0, 2\pi[. \tag{3}$$

they are obtained as the eigenvectors of the so-called monodromy matrix

$$\mathcal{M}(\omega) = \mathcal{R}(x_0 + d, x_0). \tag{4}$$

This leads directly to a generalization of Bloch waves to lossy media. Indeed, we do not have to assume that ε is real. In fact we can define the so-called Bloch variety in the following way:

$$\mathcal{B} = \{(\mu, \omega) \in \mathbb{C},\ \text{such that there exists an eigenvector } \mathcal{U} \text{ of } \mathcal{M}(\omega) \text{ with eigenvalue } \mu\} \tag{5}$$

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The relevant quantity is in fact the vector space generated by $\mathcal{U}(x_0)$ which defines a point in the projective space $\mathbb{C}P^1$.

Therefore, once the origin is chosen, we obtain a function from the Brillouin zone to $\mathbb{C}P^1$

$$k \in S^1 \longrightarrow \widetilde{\chi}(k) = [u(x_0;k) : u'(x_0;k)] \in \mathbb{C}P^1. \tag{6}$$

Two charts can be chosen to represent $\widetilde{\chi}$ as a function with values in $\mathbb{C}$: $\chi^+(k) = u(x_0;k)/u'(x_0;k)$ and $\chi^-(k) = u'(x_0;k)/u(x_0;k)$. Any complex bundle over S^1 is trivial. A non-trivial topology is found if a supplementary

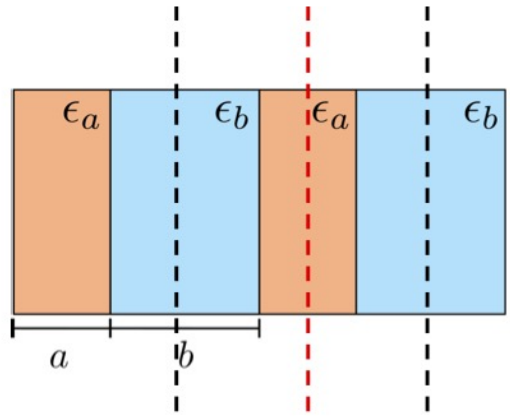

Figure 1. An example of stratified medium with inversion symmetry centers.

condition of inversion symmetry is imposed, see fig. 1. Because of the symmetry $V(x) = V(-x)$, the two Bloch eigenvectors $(\mathcal{U}, \mathcal{V})$ are related by

$$\mathcal{V} = \sigma_z \mathcal{U}, \tag{7}$$

where σ_z is the usual Pauli matrix. Denoting $z = e^{ik}$, $k \in S^1$, it holds that

$$\chi[\mathcal{U}](z) = -\chi[\mathcal{U}](1/z) = -\chi[\mathcal{V}](z) \tag{8}$$

For $z = \pm 1$ (at the boundaries of the conduction bands), either $\chi^{\pm}$ is null or it has a pole. The pole corresponds to an antisymmetric Bloch wave (Berry phase equals to π) and the zero to a symmetric one (Berry phase equals to 0). The edge modes are characterized by the following result:

Proposition 1. *Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be the monodromy matrix of each photonic crystal. For a Bloch wavevector k, there exists an eigenvector vector $U(k_0)$ defining an edge state provided the following conditions are fulfilled*

- *The matrices $\mathcal{M}_1(k)$ and $\mathcal{M}_2(k)$ have a common gap at the Bloch wavevector k, i.e. $|\mathrm{tr}(\mathcal{M}_1)| > 2$ and $|\mathrm{tr}(\mathcal{M}_1)| > 2$,*
- *the matrices $\mathcal{M}_1(k_0)$ and $\mathcal{M}_2(k)$ commute:$[M_1(k), M_2(k)] = 0$,*
- *the associated Bloch functions $\chi_1(k)$ and $\chi_2(k)$ have opposite signs.*

The advantage of considering the pole structure instead of the integral of the Berry connection over the Brillouin zone is that the pole structure evolves smoothly under a perturbation of the medium. It is not the case of the Berry connection as it is destroyed as soon as the medium is no longer periodic. The pole structure allows to study the edge state even when some randomness is added to the structure.

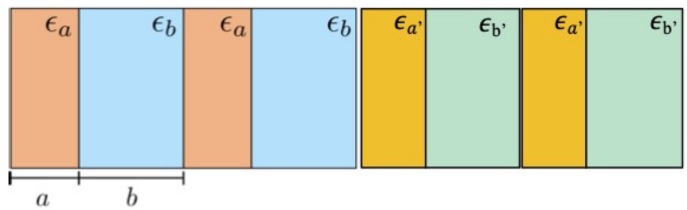

Figure 2. Two 1D photonic crystals put side by side. The random size of the layers is not shown.

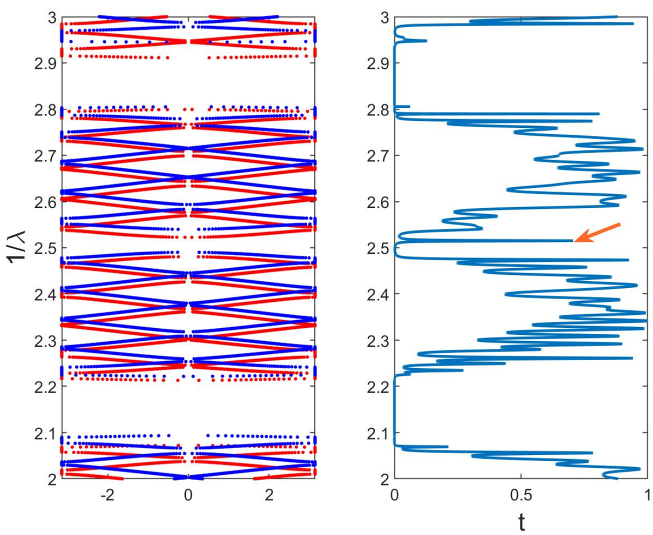

Figure 3. The functions χ for a randomly perturbed medium. It is the same as that in fig. 2 with the width of the layers modified randomly with at most 1%. An edge mode is indicated by an arrow.

3. EDGE STATES FOR A RANDOM POTENTIAL

We consider two 1D photonic crystals with different topological pole-zero structures (see fig. 2). The band structure of the each photonic crystal is given in fig. 3.

The pole structures of the corresponding functions χ are given in fig. 4. It can be seen that the band gap where the edge mode appears corresponds to a different alternance of poles and zeros for each function, thus showing that the photonic crystals belong to different topological classes.

The alternance of poles and zeros is a topological invariant. Exchanging a pole and a zero necessarily closes the gap. Poles and zeros still exist when randomness is added in the materials. The pole-zero structure is represented in fig. 4. The alternance of poles and zeros indicates that the 1D photonic crystals are in different topological states, despite the disorder.

4. EXTENSION TO 2D STRUCTURES

The poles and zeros structure associated with a Bloch mode in one dimension cannot be extended straightforwardly in 2D, because the space of solutions is not generated by a vector with two components. However, it seems possible to define topological invariant when relaxing the periodicity condition. Indeed, when edge states are considered, the structures that are involved are semi-infinite or even finite. Therefore, the topological characteristics of the fields are encoded in the finite medium. A pole structure can be defined by considering the poles of the scattering matrix of the finite medium. The range of residue of the scattering matrix at the poles gives the

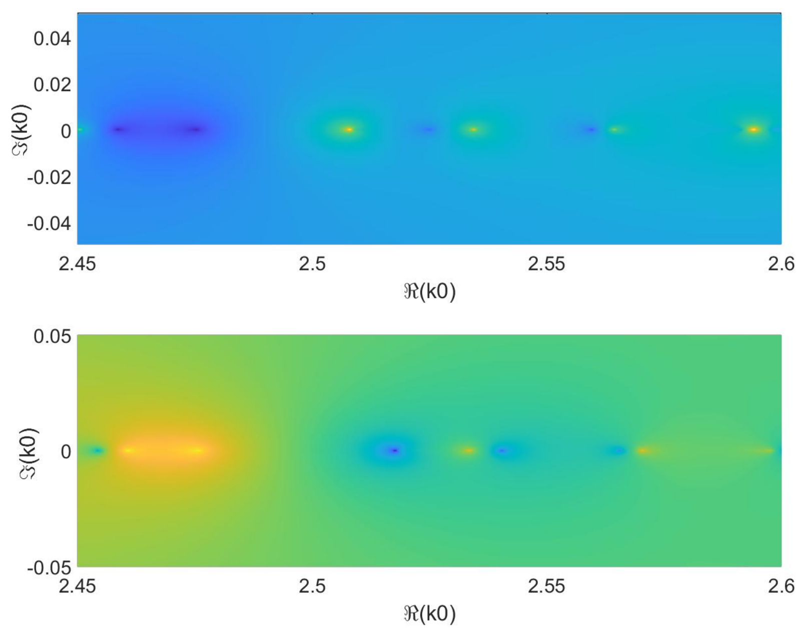

Figure 4. Pole and zero structure of the left and right photonic crystals in the presence of disorder. In the band gap between 2.5 and 2.55 the pole and zero are switched, which results in different topological classes.

eigenspace associated with the pole. In the thermodynamic limit where the medium tends to infinity, the Bloch modes are obtained. The associated poles are stable under a deformation, e.g. when introducing disorder into the structure. In doing so, the part of the modes that are linked with the symmetry of the structure are rapidly destroyed, but the part linked with the inner resonances of the scatterers is preserved. In fig. 5, the dispersion curve for a 2D photonic crystal of dielectric rods is shown. The part of the dispersion curves in the vicinity of

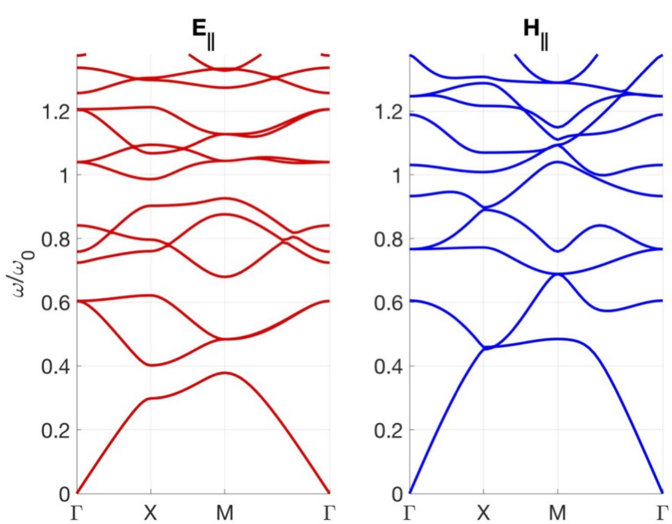

Figure 5. Band structure

$\omega/\omega_0 = 0.6$ is due to the presence of Mie resonances. When disorder is introduced inside the structure, there

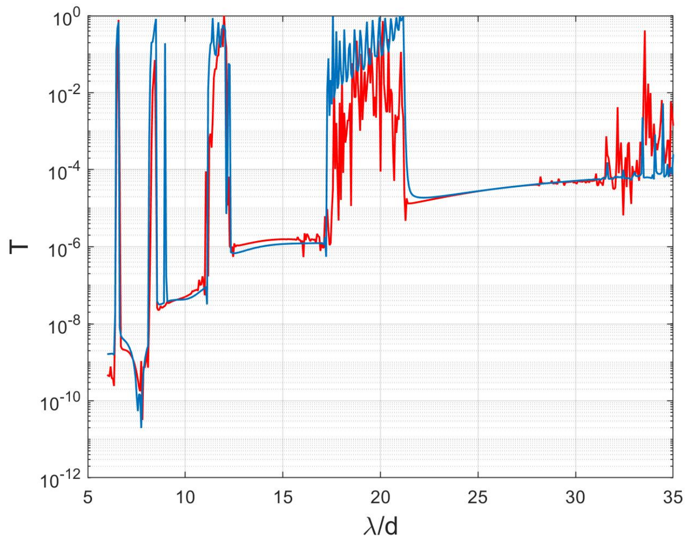

Figure 6. Transmission spectrum through the photonic crystal. In blue, the periodic structure, in red the random structure.

is a preservation of certain parts of the conduction bands. The transmission spectra through a 9×9 photonic crystal are given in fig. 6 for a periodic and a completely disordered structure.

5. CONCLUSION

We have provided a reformulation of the Zak phase in terms of meromorphic functions defined on the complex Brillouin zone. This allowed to define a topological pole-zero structure that can be extended to non-periodic structures.

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